Math 202

Quiz # 2 , May 9, 2002

Answer all questions.Do your work for problem 1 on page 1, for problem 2 on page2 and so forth. You may use the question paper for rough work. If you need extra space use space in booklet marked " for rough work only."Maximum time allowed is 60 minutes. Use of Calculators is not allowed.

1. (35 points) Use a **power series** of the form $\sum_{n=0}^{\infty} c_n x^n$ to find **one** solution of the differential equation

$$xy'' + y' - 4xy = 0.$$

You must supply explicit formulas for c_{2n} and c_{2n-1}, and prove them.

2. (35 points) Use an **extended power series** of the form $\sum_{n=0}^{\infty} c_n x^{n+r}$ to find **one** solution of the differential equation

$$x^2y'' + 2xy' + x^2y = 0.$$

You must supply an explicit formula for c_n, and prove it.

3. (30 points)The two parts in this problem are independent of each other.

- Find by the method of variation of parameters a particular solution of the differential equation

$$x^2y'' + 6xy' + 6y = x^{10}.$$

- Recall that the Bessel function J_v is defined by

$$J_v(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+v}}{2^{2n+v} n! \Gamma(n+v+1)}.$$

Prove that $(x^v J_v(x))' = x^v J_{v-1}(x)$.