

Math 202 - Exam 2 (Spring 10)

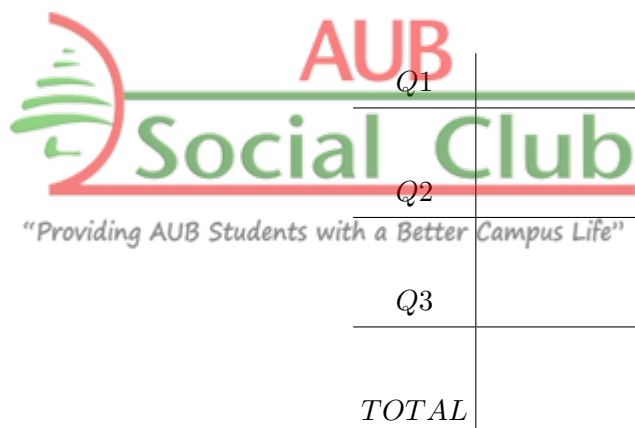
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- Please answer question 3 on the same sheet of paper on which it is written. Questions 1 and 2 have an extra sheet for you to write your answer on it. Any part of your answer written on the wrong page will not be graded.
 - There are 3 problems in total. The first question has several parts. Make sure that you attempt them all.
 - This is a closed book exam and no calculators are allowed.
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...Together At Work

Name :

ID # :



Problem 1

(15 points each) Solve the following IVPs:

i-

$$y''' - 2y'' + y' - e^x = 0 \quad ; \quad y(0) = 0 \quad , \quad y'(0) = 0 \quad , \quad y''(0) = 0$$

ii-

$$x^2 y'' + xy' - 4y = 0 \quad ; \quad y(1) = 0 \quad , \quad y'(1) = 1$$

iii-

$$y'' + y - \frac{1}{\cos(x)} = 0 \quad ; \quad y(0) = 0 \quad , \quad y'(0) = 1$$

iv-

$$y'' - y'e^y = 0 \quad ; \quad y\left(-\frac{1}{e}\right) = 1 \quad , \quad y'\left(-\frac{1}{e}\right) = e$$

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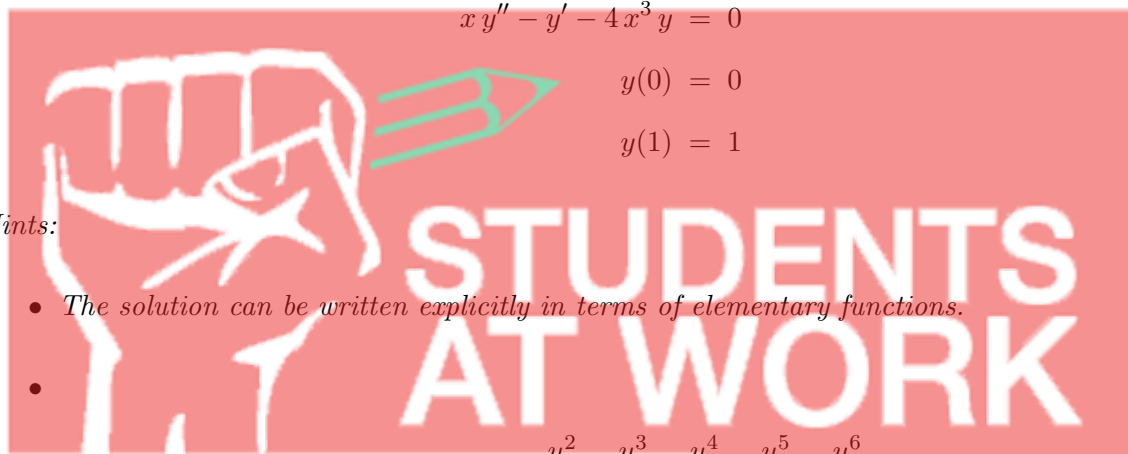


ADDITIONAL SHEET FOR PROBLEM 1 ANSWER



Problem 2

(30 points) Solve the following BVP:



$x y'' - y' - 4 x^3 y = 0$

$y(0) = 0$

$y(1) = 1$

Hints:

- The solution can be written explicitly in terms of elementary functions.
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$$e^u = 1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \frac{u^4}{4!} + \frac{u^5}{5!} + \frac{u^6}{6!} + \dots$$
$$e^{-u} = 1 - u + \frac{u^2}{2!} - \frac{u^3}{3!} + \frac{u^4}{4!} - \frac{u^5}{5!} + \frac{u^6}{6!} + \dots$$

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ADDITIONAL SHEET FOR PROBLEM 2 ANSWER



Problem 3

(10 points) Show that

$$xJ'_\nu(x) = \nu J_\nu(x) - xJ_{\nu+1}(x)$$

where $J_\nu(x)$ is the Bessel function of the first kind of order ν and is given by the following series

$$J_\nu(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \Gamma(1 + \nu + n)} \left(\frac{x}{2}\right)^{2n+\nu}$$

