Math 202 — Spring 2011
Differential Equations, sections 16–19
Quiz 2, April 20 — Duration: 1 hour

REMINDER — PLEASE READ THE INSTRUCTIONS BELOW FIRST

---

1. Write your name, AUB ID number, and section number ON THE FRONT COVER OF YOUR AUB EXAMINATION BOOKLET.
To remind you, the sections are as follows:

Section 16	Section 17	Section 18	Section 19
Recitation Tu 12:30	Recitation Tu 3:30	Recitation Tu 5	Recitation Tu 2
Prof. Makdisi	Ms. Jaber	Ms. Jaber	Ms. Jaber

2. You may work on the problems in ANY ORDER in your exam booklet, but please make it clear which problem you are solving on any given page. In particular, PLEASE INDICATE IF THE SOLUTION TO A PROBLEM IS CONTINUED ON A LATER PAGE.

3. Do as much of the exam as you can, and budget your time carefully. Take a minute at the start of the exam to decide which problems to work on first.

4. Explain your steps precisely and clearly to ensure full credit. Partial solutions will receive partial credit. Each problem is worth 11 points, and there are 7 problems for a TOTAL of 77 points.

5. Do NOT bother to simplify final answers; if an answer is something like $y = Ae^x + e^x \cdot (\ln(1+\sin x) - \ln(1-\sin x))/2$, just leave it that way. Also, if you cannot do a certain integral, leave it as an integral in your solution for partial credit on the rest of the problem. YOU MAY LEAVE THE SOLUTION TO A DIFFERENTIAL EQUATION IN IMPLICIT FORM.

6. No calculators, books, or notes allowed. Turn off and put away any cell phones.

GOOD LUCK!

(Remember, each problem is worth 11 points, for a total of 77 points)

1 (11 pts). Find the general solution of $y' = \dfrac{(x+y)(x+y-1)}{(x+y)^3+2} - 1$.

2 (11 pts). Solve the initial-value problem $3xy' + (3x+1)y = \dfrac{x^2}{y^2}, \quad y(1/3) = 1$.

3. Consider a differential equation $M\,dx + N\,dy = 0$.

a) **(3 pts)** Suppose that one can find an integrating factor of the form $\mu = \mu(x)$ for the above equation. Prove that μ must satisfy a differential equation of the form $\dfrac{d\mu/dx}{\mu}$ = something in terms of M and N. **DO NOT just write down a memorized formula for μ! Be sure to justify your work.**

b) **(3 pts)** Why does the equation $x^2y^2\,dx + (x+y)dy = 0$ **NOT** have an integrating factor of the form $\mu = \mu(x)$? Justify.

c) **(5 pts)** Find the general solution of $(2xy + y + 2y^3)dx + (x + 3y^2)dy = 0$.

4 (11 pts). Find the general solution of $y'' - 2y' + 2y = 2x^2 - 4x + \dfrac{e^x}{\cos^3 x}$.

5. a) **(4 pts)** Let $a \neq 0$. Show that the functions $z_1 = 1, z_2 = x, z_3 = e^{ax}$ are linearly independent. What happens if $a = 0$?

b) **(4 pts)** Find three solutions y_1, y_2, y_3 of the differential equation $y''' + 4y'' - 3y' - 18y = 0$ and **use them to write down** the general solution.

c) **(3 pts)** Prove that the functions y_1, y_2, y_3 from part (b) are linearly independent. (Hint: to save time, use part (a) in a suitable way.)

6. a) **(9 pts)** Solve the initial value problem $x^2y'' - 6xy' + 10y = x^4, \quad y(1) = y'(1) = 1$.

b) **(2 pts)** What changes if the initial conditions are $y(0) = y'(0) = 0$? Explain.

7 (11 pts). Find the general solution of the equation $y'' - x^2y' - 2xy = 0$, using a **power series** centered at $x = 0$. You need only give the first four nonzero terms of y_1 and y_2. (Hint: this involves going up to x^{10}.)

STUDENTS AT WORK
...Together At Work
AUB Social Club
"Providing AUB Students with a Better Campus Life"